# Data Exchange beyond Complete Data

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# Outline

- The need for a more general data exchange framework
  - Two important scenarios: Incomplete databases and knowledge bases
- Formalism for exchanging representations systems
- Applications to incomplete databases
- Applications to metadata management
- Concluding remarks

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# Data exchange: Some lessons learned

Key steps in the development of the area:

- Definition of schema mapping: Precise syntax and semantics
- Definition of the notion of solution
- Identification of good solutions
  - Universal solutions
- Polynomial time algorithms for materializing good solutions
  - Based on the chase procedure

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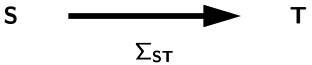
  - Based on the chase procedure

Creating schema mappings is a time consuming and expensive process

- Manual or semi-automatic process in general

# Ongoing project: Reusing schema mappings



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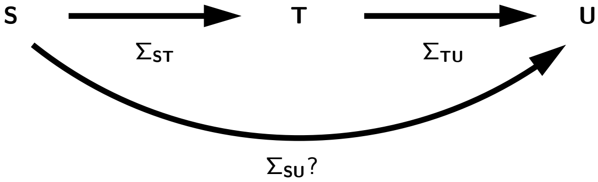

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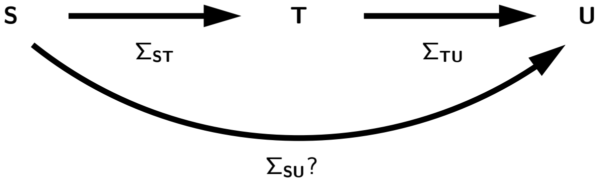


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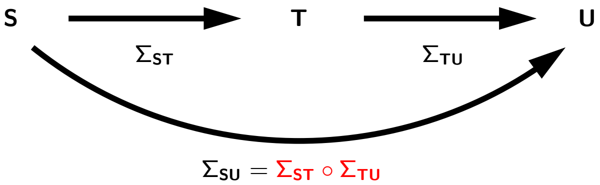



We need some operators for schema mappings

- **Composition** in the above case

# Metadata management

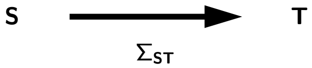
This has motivated the need for the development of a general infrastructure for managing schema mappings.

The problem of managing schema mappings is called **metadata management.**

High-level algebraic operators, such as compose, are used to manipulate mappings.

- What other operators are needed?

# An inverse operator is also needed



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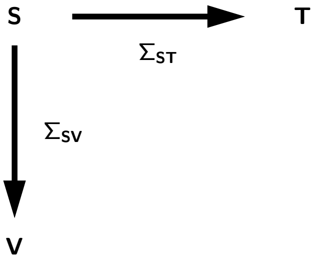

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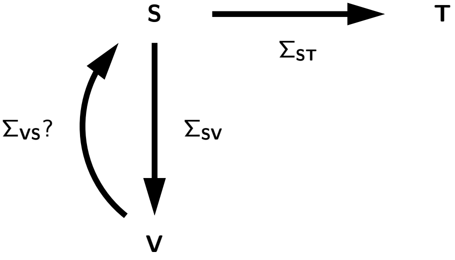

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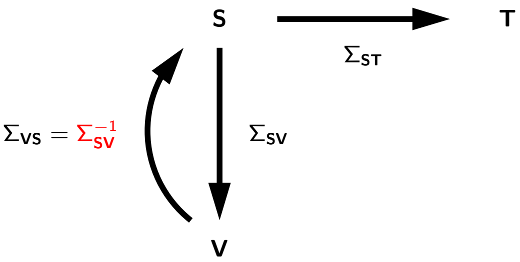

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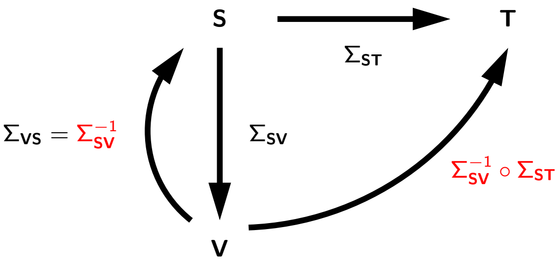


Composition and inverse operators have to be combined

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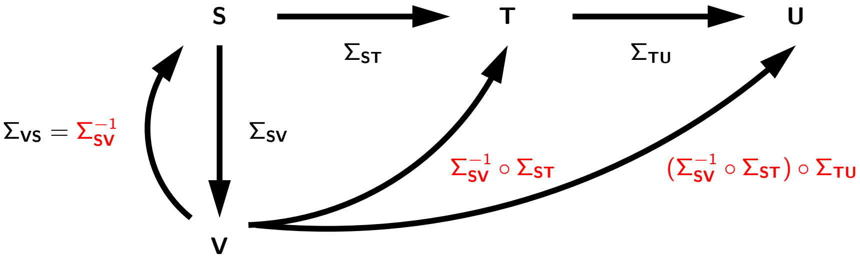


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# Metadata management: A more general data exchange framework is needed

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- Sources instances may contain null values

There is a need for a data exchange framework that can handle databases with incomplete information.

# Data exchange in the RDF world

There is an increasing interest in publishing relational data as RDF

- Resulted in the creation of the W3C RDB2RDF Working Group

The problem of translating relational data into RDF can be seen as a data exchange problem

- Schema mappings can be used to describe how the relational data is to be mapped into RDF

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But there is a mismatch here: A relational database under a closed-world semantics is to be translated into an RDF graph under an open-world semantics

- There is a need for a data exchange framework that can handle both databases with complete and incomplete information

# Data exchange in the RDF world

An issue discussed at the W3C RDB2RDF Working Group: Is a mapping information preserving?

- In particular: For the default mapping defined by this group

How can we address this issue?

- Metadata management can help us

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Question to answer: Is a mapping invertible?

- This time an RDF graph is to be translated into a relational database!
- We want to have a unifying framework for all these cases

# But these are not the only reasons . . .

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- A knowledge base has not only data but also rules that allows to infer new data
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> The input is a mapping and a source specification including data and rules, and the output is a target specification also including data and rules

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There is a need for a data exchange framework that can handle knowledge bases.

# One can exchange more than complete data

- In data exchange one starts with a database instance (with complete information).
- What if we have an initial object that has several interpretations?
  - A representation of a set of possible instances
- We propose a new general formalism to exchange representations of possible instances
  - We apply it to the problems of exchanging instances with incomplete information and exchanging knowledge bases

# Outline

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# Representation systems

A representation system $\mathcal{R} = (\mathbf{W}, \text{rep})$ consists of:

- a set **W** of *representatives*
- a function rep that assigns a set of instances to every element in **W**

$$\text{rep}(\mathcal{V}) = \{I_1, I_2, I_3, \ldots\} \text{ for every } \mathcal{V} \in \mathbf{W}$$

Uniformity assumption: For every $\mathcal{V} \in \mathbf{W}$, there exists a relational schema **S** (the type of $\mathcal{V}$) such that $\text{rep}(\mathcal{V}) \subseteq \text{Inst}(\mathbf{S})$

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Incomplete instances and knowledge bases are representation systems

# In classical data exchange we consider only *complete* data

$\mathcal{M}$ is a mapping from **S** to **T** if $\mathcal{M} \subseteq \text{Inst}(\mathbf{S}) \times \text{Inst}(\mathbf{T})$

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$\mathcal{M}$ is defined by a set $\Sigma$ of dependencies (e.g., st-tgds) if: $(I, J) \in \mathcal{M}$ iff $(I, J) \models \Sigma$.

- Notation: $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$

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This can be extended to set of instances. Given $\mathcal{X} \subseteq \mathsf{Inst}(\mathbf{S})$:

$$\mathsf{Sol}_{\mathcal{M}}(\mathcal{X}) = \bigcup_{I \in \mathcal{X}} \mathsf{Sol}_{\mathcal{M}}(I)$$

# Extending the definition to representation systems

Given:

- a mapping $\mathcal{M}$ from **S** to **T**
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**Definition (APR11)**

$\mathcal{V}$ is an $\mathcal{R}$-*solution* of $\mathcal{U}$ under $\mathcal{M}$ if

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Or equivalently: $\mathcal{V}$ is an $\mathcal{R}$-*solution* of $\mathcal{U}$ if for every $J \in \text{rep}(\mathcal{V})$, there exists $I \in \text{rep}(\mathcal{U})$ such that $J \in \text{Sol}_{\mathcal{M}}(I)$.

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$$\mathcal{U} \xrightarrow{\quad\quad\quad\mathcal{M}\quad\quad\quad} \mathcal{V}$$

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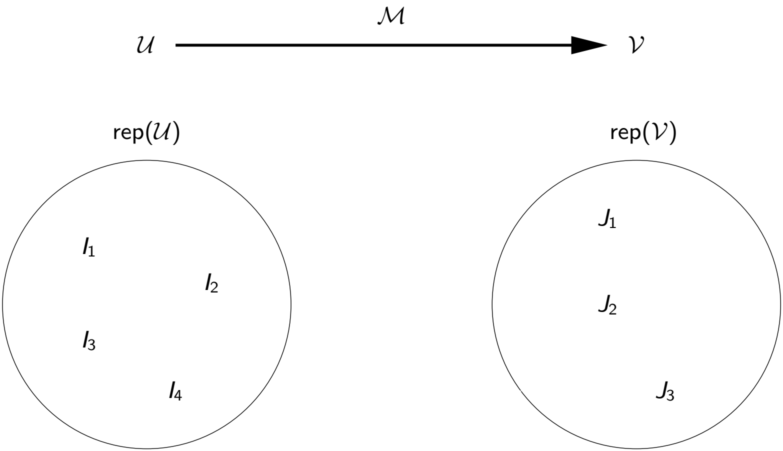

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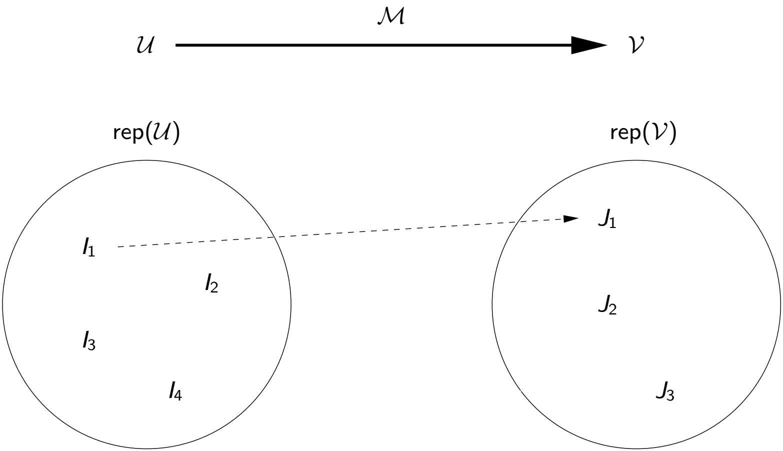

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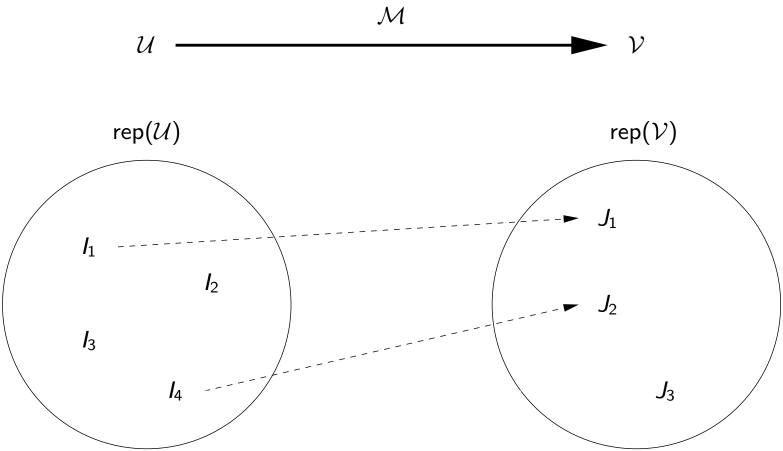

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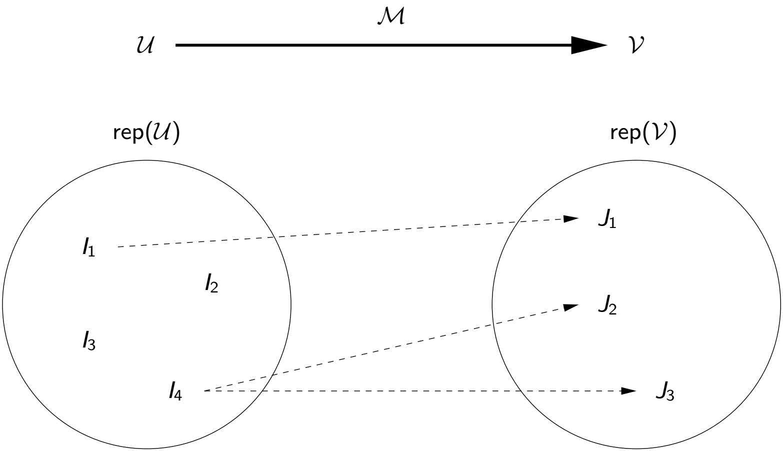

# Universal solutions

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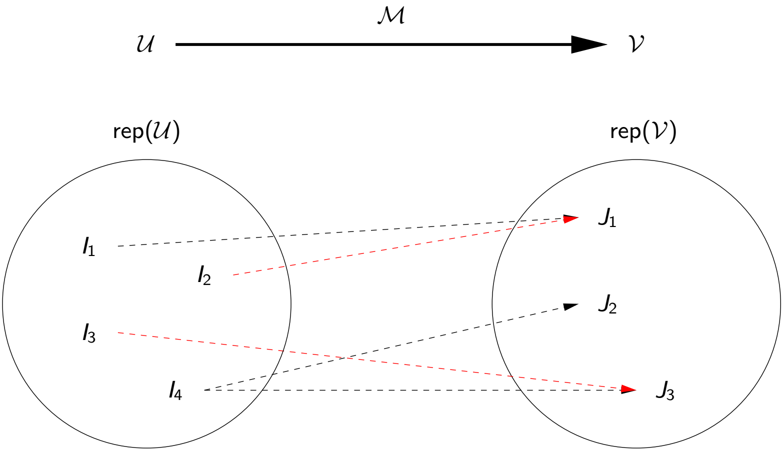
**Definition (APR11)**

$\mathcal{V}$ is an *universal $\mathcal{R}$-solution* of $\mathcal{U}$ under $\mathcal{M}$ if

$$\text{rep}(\mathcal{V}) \;=\; \text{Sol}_{\mathcal{M}}(\text{rep}(\mathcal{U}))$$

# Universal solutions in a figure



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$\mathcal{R} = (\mathbf{W}, \text{rep})$ is a *strong representation system* for $\mathcal{C}$ if for every $\mathcal{M} \in \mathcal{C}$ and for every $\mathcal{U} \in \mathbf{W}$, there exists a $\mathcal{V} \in \mathbf{W}$:

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If $\mathcal{R} = (\mathbf{W}, \text{rep})$ is a strong representation system, then the universal solutions for the representatives in $\mathbf{W}$ can be represented in the same system.

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# Motivating questions

What is a strong representation system for the class of mappings specified by st-tgds?

- Are instances including nulls enough?

Can the fundamental data exchange problems be solved in polynomial time in this setting?

- Computing (universal) solutions

# Naive instances

We have already considered naive instances: Instances with null values

- Example: Universal solutions

A naive instance $\mathcal{I}$ has labeled nulls:

$$R(1, n_1)$$
$$R(n_1, 2)$$
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The interpretations of $\mathcal{I}$ are constructed by replacing nulls by constants:

$$\text{rep}(\mathcal{I}) \quad = \quad \{K \mid \mu(\mathcal{I}) \subseteq K \text{ for some valuation } \mu\}$$

# Are naive instances expressive enough?

Naive instances have been extensively used in data exchange:

**Proposition (FKMP03)**

*Let* $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$*, where* $\Sigma$ *is a set of st-tgds. Then for every instance* $I$ *of* $\mathbf{S}$*, there exists a naive instance* $\mathcal{J}$ *of* $\mathbf{T}$ *such that:*

$$rep(\mathcal{J}) \quad = \quad \mathrm{Sol}_{\mathcal{M}}(I)$$

In fact, every universal solution satisfies the property mentioned above.

# Are naive instances expressive enough?

But naive instances are not expressive enough to deal with incomplete information in the source instances:

**Proposition (APR11)**

*Naive instances are not a strong representation system for the class of mappings specified by st-tgds*

# Are naive instances expressive enough?

## Example

Consider a mapping $\mathcal{M}$ specified by:

$$\begin{aligned}\texttt{Manager}(x, y) &\rightarrow \texttt{Reports}(x, y)\\ \texttt{Manager}(x, x) &\rightarrow \texttt{SelfManager}(x)\end{aligned}$$

The *canonical* universal solution for $\mathcal{I} = \{\texttt{Manager}(n, \text{Peter})\}$ under $\mathcal{M}$:

$$\mathcal{J} = \{\texttt{Reports}(n, \text{Peter})\}$$

But $\mathcal{J}$ is not a *good* solution for $\mathcal{I}$.

- It cannot represent the fact that if $n$ is given value Peter, then $\texttt{SelfManager}(\text{Peter})$ should hold in the target.

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Conditional instances: Naive instances plus *tuple conditions*

A tuple condition is a positive Boolean combinations of:

- equalities and inequalities between nulls, and between nulls and constants

# Conditional instances

## Example

$$\begin{array}{l|l} R(1, n_1) & n_1 = n_2 \\ R(n_1, n_2) & n_1 \neq n_2 \ \vee \ n_2 = 2 \end{array}$$

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Semantics:

$\underline{\mu(n_1) = \mu(n_2) = 2}$ $\underline{\mu(n_1) = \mu(n_2) = 3}$ $\underline{\mu(n_1) = 2, \mu(n_2) = 3}$

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| | |
|---|---|
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Interpretations of a conditional instance $\mathcal{I}$:

$$\text{rep}(\mathcal{I}) \quad = \quad \{K \mid \mu(\mathcal{I}) \subseteq K \text{ for some valuation } \mu\}$$

# Positive conditional instances

Many problems are intractable over conditional instances.

- We also consider a restricted class of conditional instances

Positive conditional instances: Conditional instances without inequalities

# (Positive) conditional instances are enough

## Theorem (APR11)

*Both conditional instances and positive conditional instances are strong representation systems for the class of mappings specified by st-tgds.*

## Example

Consider again the mapping $\mathcal{M}$ specified by:

$$\begin{array}{rcl} \texttt{Manager}(x, y) & \rightarrow & \texttt{Reports}(x, y) \\ \texttt{Manager}(x, x) & \rightarrow & \texttt{SelfManager}(x) \end{array}$$

The following is a universal solution for $\mathcal{I} = \{\texttt{Manager}(n, \text{Peter})\}$

$$\begin{array}{l|l} \texttt{Reports}(n, \text{Peter}) & \textit{true} \\ \texttt{SelfManager}(\text{Peter}) & n = \text{Peter} \end{array}$$

# Positive conditional instances are *exactly* the needed representation system

### Theorem (APR11)

*All the following are needed to obtain a strong representation system for the class of mappings specified by st-tgds:*

# Positive conditional instances are *exactly* the needed representation system

**Theorem (APR11)**

*All the following are needed to obtain a strong representation system for the class of mappings specified by st-tgds:*

- *equalities between nulls*

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Conditional instances are enough but not minimal.

# Positive conditional instance can be used in practice!

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*There exists a polynomial time algorithm that, given a positive conditional instance $\mathcal{I}$ over $\mathbf{S}$, computes a positive conditional instance $\mathcal{J}$ over $\mathbf{T}$ that is a universal solution for $\mathcal{I}$ under $\mathcal{M}$.*

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**Remark**

They are also appropriate for query answering in data exchange.

- Same polynomial-time cases as in the usual setting

# Outline

- The need for a more general data exchange framework
  - Two important scenarios: Incomplete databases and knowledge bases
- Formalism for exchanging representations systems
- Applications to incomplete databases
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- Concluding remarks

# The composition operator

**Definition (FKPT04)**

Let $\mathcal{M}_{12}$ be a mapping from $\mathbf{S}_1$ to $\mathbf{S}_2$, and $\mathcal{M}_{23}$ a mapping from $\mathbf{S}_2$ to $\mathbf{S}_3$:

$$\mathcal{M}_{12} \circ \mathcal{M}_{23} = \{(I_1, I_3) \mid \exists I_2 : (I_1, I_2) \in \mathcal{M}_{12} \text{ and } (I_2, I_3) \in \mathcal{M}_{23}\}$$

# Expressing the composition of mappings

**Question**

What is the right language for expressing the composition?

- st-tgds?

**Example (FKPT04)**

Consider the mappings $\mathcal{M}_{12}$:

$$\begin{aligned} node(x) &\rightarrow \exists y\, coloring(x, y) \\ edge(x, y) &\rightarrow edge'(x, y) \end{aligned}$$

and $\mathcal{M}_{23}$:

$$\begin{aligned} edge'(x, y) \wedge coloring(x, u) \wedge coloring(y, u) &\rightarrow error(x, y) \\ coloring(x, y) &\rightarrow color(y) \end{aligned}$$

# SO tgds: The right language for expressing the composition of mappings

## Example (Cont'd)

The following dependency defines the composition:

$$\exists f \Big[\forall x(node(x) \rightarrow color(f(x))) \wedge \\ \forall x \forall y(edge(x,y) \wedge f(x) = f(y) \rightarrow error(x,y))\Big]$$

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This example shows the main ingredients of SO tgds:

- Predicates including terms: $color(f(x))$
- Equality between terms: $f(x) = f(y)$

# SO tgds: The right language for expressing the composition of mappings

SO tgds were introduced in [FKPT04]

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- *There exists an exponential time algorithm that computes such SO tgds*

# SO tgds: The right language for expressing the composition of mappings

**Corollary (FKPT04)**

*The composition of a finite number of mappings, each defined by a finite set of st-tgds, is defined by an SO tgd*

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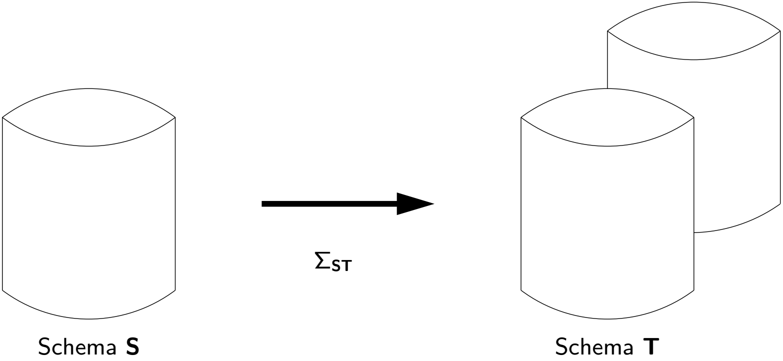
*The composition of a finite number of mappings, each defined by a finite set of st-tgds, is defined by an SO tgd*

But not only that, SO tgds are *exactly* the right language:

**Theorem (FKPT05)**

*Every SO tgd defines the composition of a finite number of mappings, each defined by a finite set of st-tgds.*

# The inverse operator



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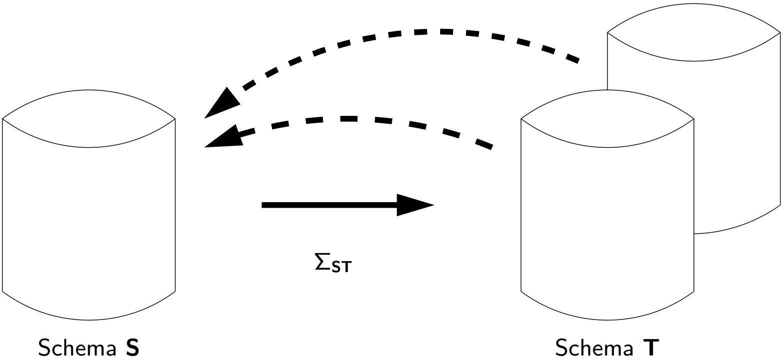

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What is the semantics of the inverse operator?

This turns out to be a very difficult question.

Several notions of inverse have been considered:

- Fagin-inverse [F06]
- Quasi-inverse [FKPT07]
- Maximum recovery [APR08]
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# Recovery: specifies how to recover sound information

Data may be lost in the exchange through a mapping $\mathcal{M}$

- We would like to find a mapping $\mathcal{M}^\star$ that at least recovers sound data w.r.t. $\mathcal{M}$
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Consider a mapping $\mathcal{M}$ specified by:

$$emp(x, y, z) \wedge y \neq z \quad \rightarrow \quad shuttle(x, z)$$

What mappings are recoveries of $\mathcal{M}$?

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We would like to find a recovery of $\mathcal{M}$ that is better than any other recovery: Maximum recovery

# The notion of recovery: Formalization

**Definition (APR08)**

Let $\mathcal{M}$ be a mapping from $\mathbf{S}_1$ to $\mathbf{S}_2$ and $\mathcal{M}^\star$ a mapping from $\mathbf{S}_2$ to $\mathbf{S}_1$. Then $\mathcal{M}^\star$ is a recovery of $\mathcal{M}$ if:

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**Example**

Consider again mapping $\mathcal{M}$ specified by:

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This mapping is not a recovery of $\mathcal{M}$:

$$\mathcal{M}_3^\star\text{:} \quad shuttle(x, z) \quad \rightarrow \quad \exists u\, emp(x, z, u)$$

# The notion of recovery: Formalization

## Example (Cont'd)

On the other hand, these mappings are recoveries of $\mathcal{M}$:

$$
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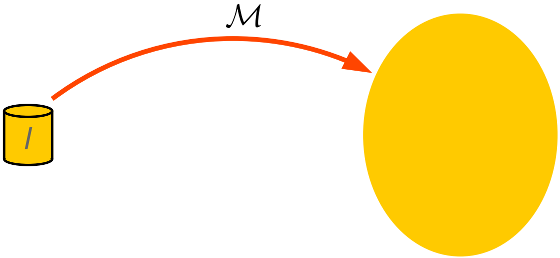

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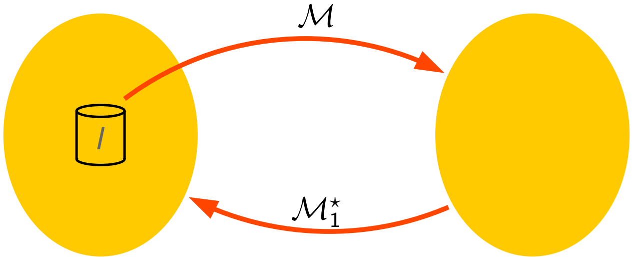

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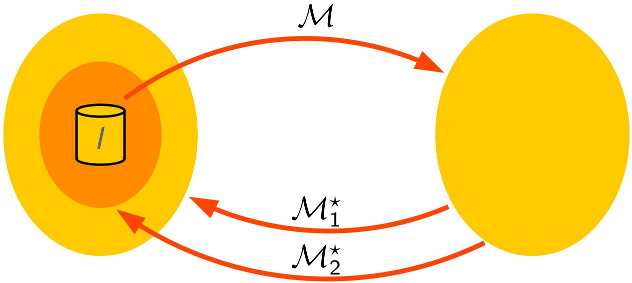

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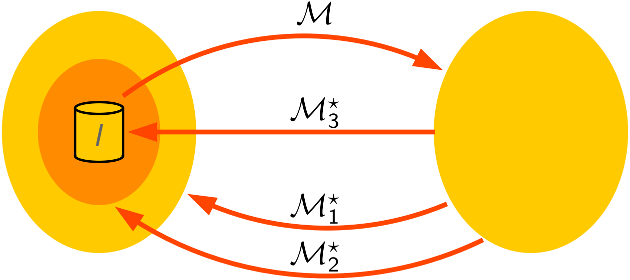

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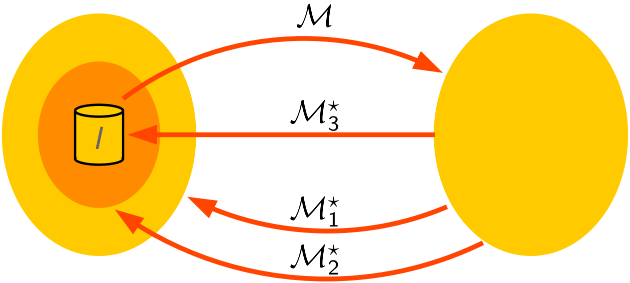


**Definition (APR08)**

$\mathcal{M}^\star$ is a maximum recovery of $\mathcal{M}$ if:

- $\mathcal{M}^\star$ is a recovery of $\mathcal{M}$
- for every recovery $\mathcal{M}'$ of $\mathcal{M}$: $\mathcal{M} \circ \mathcal{M}^\star \subseteq \mathcal{M} \circ \mathcal{M}'$

# On the existence of maximum recoveries

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*Every mapping specified by a finite set of st-tgds has a maximum recovery.*

But this does not hold if one also considers naive instances in the source.

- Maximum extended recovery was introduced to overcome this limitation

# We need to combine the operators

Can we combine the composition and inverse operators?

- Is there a good language for both operators?

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Can we combine the composition and inverse operators?

- Is there a good language for both operators?

Bad news:

**Theorem (APR11)**

*There exists a mapping specified by an SO tgd that does not have a maximum recovery.*

# We need to combine the operators

Even worse:

- Previous mapping has neither a Fagin-inverse nor a quasi-inverse nor a $\mathcal{C}$-maximum recovery ($\mathbf{CQ} \subseteq \mathcal{C}$)
- Semantics of maximum extended recovery is appropriate for st-tgds.

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- No, we need to revisit the semantics of mappings

# What went wrong?

Key observation: A target instance of a mapping can be the source instance of another mapping.

- Sources instances may contain null values

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**Theorem (APR11)**

*Positive conditional instances are a strong representation system for the class of mappings specified by SO tgds.*

# A solution to the problem

**Theorem (APR11)**

*If (usual) instances are replaced by positive conditional instances:*

- *SO tgds are still the right language for the composition of mappings given by st-tgds*
- *Every mapping specified by an SO tgd admits a maximum recovery*

# Outline

- The need for a more general data exchange framework
  - Two important scenarios: Incomplete databases and knowledge bases
- Formalism for exchanging representations systems
- Applications to incomplete databases
- Applications to metadata management
- Concluding remarks

# We can exchange more than complete data

We propose a general formalism to exchange *representation systems*

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Next step: Apply our general setting to the Semantic Web

- Semantic Web data has *nulls* (blank nodes)
- Semantic Web specifications have rules (RDFS, OWL)

# Thank you!

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