

# Exchanging more than Complete Data

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# Outline: First part

- ▶ The data exchange problem
  - ▶ Some fundamental results in relational data exchange
- ▶ The need for a more general data exchange framework
  - ▶ Two important scenarios: Incomplete databases and knowledge bases

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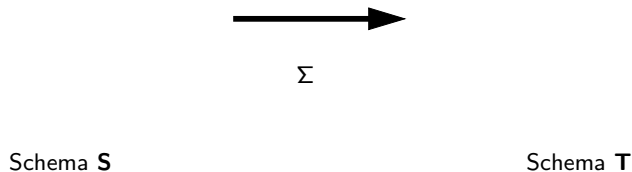
# The problem of data exchange

Given: A source schema **S**, a target schema **T** and a specification  $\Sigma$  of the relationship between these schemas

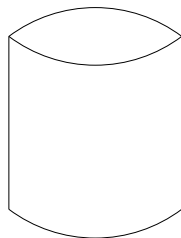
Data exchange: Problem of materializing an instance of **T** given an instance of **S**

- ▶ Target instance should reflect the source data as accurately as possible, given the constraints imposed by  $\Sigma$  and **T**
- ▶ It should be efficiently computable
- ▶ It should allow one to evaluate queries on the target in a way that is *semantically consistent* with the source data

# Data exchange in a picture



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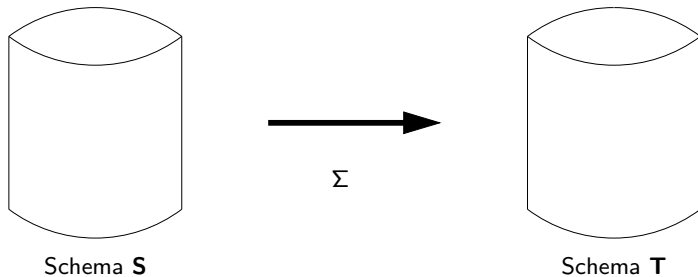
Schema **S**



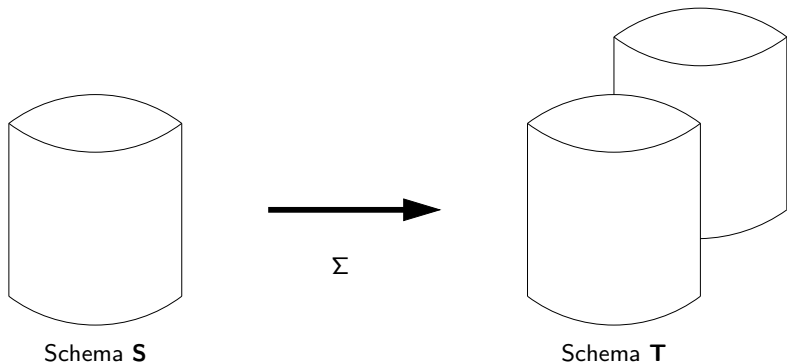
$\Sigma$

Schema **T**

# Data exchange in a picture

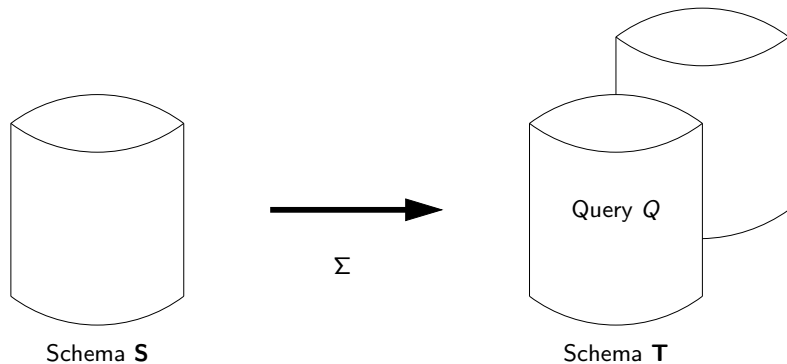


# Data exchange in a picture





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# Data exchange: Some fundamental questions

What are the challenges in the area?

- ▶ What is a good language for specifying the relationship between source and target data?
  - ▶ Expressiveness versus complexity
- ▶ What is a good instance to materialize?
- ▶ What does it mean to answer a query over target data?
- ▶ How do we answer queries over target data? Can we do this efficiently?

# Exchanging relational data

The data exchange problem has been extensively studied in the relational world.

- ▶ It has also been commercially implemented: IBM Clio

Relational data exchange setting:

- ▶ Source and target schemas: Relational schemas
- ▶ Relationship between source and target schemas:  
Source-to-target tuple-generating dependencies (st-tgds)

Semantics of data exchange has been precisely defined.

- ▶ Efficient algorithms for materializing target instances and for answering queries over the target schema have been developed

# Schema mapping: The key component in relational data exchange

Schema mapping:  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$

- ▶  $\mathbf{S}$  and  $\mathbf{T}$  are disjoint relational schemas
- ▶  $\Sigma$  is a finite set of st-tgds:

$$\forall \bar{x} \forall \bar{y} (\varphi(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \psi(\bar{x}, \bar{z}))$$

$\varphi(\bar{x}, \bar{y})$ : conjunction of relational atomic formulas over  $\mathbf{S}$

$\psi(\bar{x}, \bar{z})$ : conjunction of relational atomic formulas over  $\mathbf{T}$

# Relational schema mappings: An example

## Example

- ▶ **S**: Employee(name)
- ▶ **T**: Dept(name, number)
- ▶  $\Sigma$ :

$$\forall x \left( \text{Employee}(x) \rightarrow \exists y \text{Dept}(x, y) \right)$$

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## Note

We omit universal quantifiers in st-tgds:

$$\text{Employee}(x) \rightarrow \exists y \text{Dept}(x, y)$$

# Relational data exchange problem

Fixed:  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$

Problem: Given instance  $I$  of  $\mathbf{S}$ , find an instance  $J$  of  $\mathbf{T}$  such that  $(I, J)$  satisfies  $\Sigma$

- ▶  $(I, J)$  satisfies  $\varphi(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \psi(\bar{x}, \bar{z})$  if whenever  $I$  satisfies  $\varphi(\bar{a}, \bar{b})$ , there is a tuple  $\bar{c}$  such that  $J$  satisfies  $\psi(\bar{a}, \bar{c})$

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## Notation

$J$  is a **solution** for  $I$  under  $\mathcal{M}$

- ▶  $\text{Sol}_{\mathcal{M}}(I)$ : Set of solutions for  $I$  under  $\mathcal{M}$



# The notion of solution: Example

## Example

- ▶ **S**: Employee(name)
- ▶ **T**: Dept(name, number)
- ▶  $\Sigma$ : Employee( $x$ )  $\rightarrow$   $\exists y$  Dept( $x, y$ )

Solutions for  $I = \{\text{Employee(Peter)}\}$ :

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$J_4$ : {Dept(Peter,  $n_1$ )}

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Solutions for  $I = \{\text{Employee(Peter)}\}$ :

$J_1$ : {Dept(Peter,1)}

$J_2$ : {Dept(Peter,1), Dept(Peter,2)}

$J_3$ : {Dept(Peter,1), Dept(John,1)}

$J_4$ : {Dept(Peter,  $n_1$ )}

$J_5$ : {Dept(Peter,  $n_1$ ), Dept(Peter,  $n_2$ )}

# Canonical universal solution

## Algorithm

Input :  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$  and an instance  $I$  of  $\mathbf{S}$

Output : Canonical universal solution  $J^*$  for  $I$  under  $\mathcal{M}$

**let**  $J^* :=$  empty instance of  $\mathbf{T}$

**for every**  $\varphi(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \psi(\bar{x}, \bar{z})$  in  $\Sigma$  **do**

**for every**  $\bar{a}, \bar{b}$  such that  $I$  satisfies  $\varphi(\bar{a}, \bar{b})$  **do**

        create a fresh tuple  $\bar{n}$  of pairwise distinct null values

        insert  $\psi(\bar{a}, \bar{n})$  into  $J^*$

# Canonical universal solution: Example

## Example

Consider mapping  $\mathcal{M}$  specified by dependency:

$$\text{Employee}(x) \rightarrow \exists y \text{Dept}(x, y)$$

Canonical universal solution for

$I = \{\text{Employee}(\text{Peter}), \text{Employee}(\text{John})\}$ :

- ▶ For  $a = \text{Peter}$  do
  - ▶ Create a fresh null value  $n_1$
  - ▶ Insert  $\text{Dept}(\text{Peter}, n_1)$  into  $J^*$
- ▶ For  $a = \text{John}$  do
  - ▶ Create a fresh null value  $n_2$
  - ▶ Insert  $\text{Dept}(\text{John}, n_2)$  into  $J^*$

Result:  $J^* = \{\text{Dept}(\text{Peter}, n_1), \text{Dept}(\text{John}, n_2)\}$



# Query answering in data exchange

Given: Mapping  $\mathcal{M}$ , source instance  $I$  and query  $Q$  over the target schema

- ▶ What does it mean to answer  $Q$ ?

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## Definition (Certain answers)

$$\text{certain}_{\mathcal{M}}(Q, I) = \bigcap_{J \text{ is a solution for } I \text{ under } \mathcal{M}} Q(J)$$

## Example

Consider mapping  $\mathcal{M}$  specified by:

$$\text{Employee}(x) \rightarrow \exists y \text{Dept}(x, y)$$

Given instance  $I = \{\text{Employee}(\text{Peter})\}$ :

$$\begin{aligned} \text{certain}_{\mathcal{M}}(\exists y \text{Dept}(x, y), I) &= \{\text{Peter}\} \\ \text{certain}_{\mathcal{M}}(\text{Dept}(x, y), I) &= \emptyset \end{aligned}$$

# Query rewriting: An approach for answering queries

How can we compute certain answers?

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Approach proposed in [FKMP03]: **Query Rewriting**

Given a mapping  $\mathcal{M}$  and a target query  $Q$ , compute a query  $Q^*$  such that for every source instance  $I$  with canonical universal solution  $J^*$ :

$$\text{certain}_{\mathcal{M}}(Q, I) = Q^*(J^*)$$

# Query rewriting over the canonical universal solution

## Theorem (FKMP03)

*Given a mapping  $\mathcal{M}$  specified by st-tgds and a union of conjunctive queries  $Q$ , there exists a query  $Q^*$  such that for every source instance  $I$  with canonical universal solution  $J^*$ :*

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**Proof idea:** Assume that  $\mathbf{C}(a)$  holds whenever  $a$  is a constant.

Then:

$$Q^*(x_1, \dots, x_m) = \mathbf{C}(x_1) \wedge \dots \wedge \mathbf{C}(x_m) \wedge Q(x_1, \dots, x_m)$$

# Computing certain answers: Complexity

Data complexity: Data exchange setting and query are considered to be fixed.

## Corollary (FKMP03)

*For mappings given by st-tgds, certain answers for **UCQ** can be computed in polynomial time (data complexity)*



# Relational data exchange: Some lessons learned

Key steps in the development of the area:

- ▶ Definition of schema mappings: Precise syntax and semantics
  - ▶ Definition of the notion of solution
- ▶ Identification of good solutions
- ▶ Polynomial time algorithms for materializing good solutions
- ▶ Definition of target queries: Precise semantics
- ▶ Polynomial time algorithms for computing certain answers for **UCQ**

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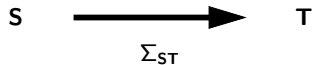
Creating schema mappings is a time consuming and expensive process

- ▶ Manual or semi-automatic process in general

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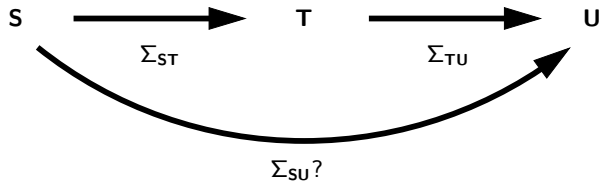
# Ongoing project: Reusing schema mappings



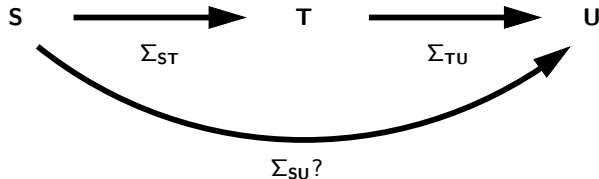
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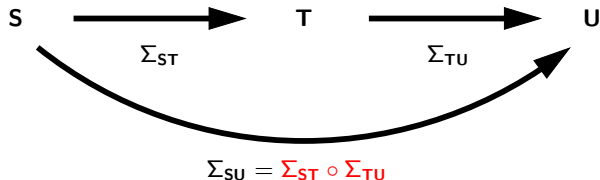


# Ongoing project: Reusing schema mappings



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We need some operators for schema mappings

- ▶ **Composition** in the above case



Contributions mentioned in the previous slides are just a first step towards the development of a general framework for data exchange.

In fact, as pointed in [B03],

many information system problems involve not only the design and integration of complex application artifacts, but also their subsequent manipulation.

# Metadata management

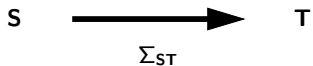
This has motivated the need for the development of a general infrastructure for managing schema mappings.

The problem of managing schema mappings is called **metadata management**.

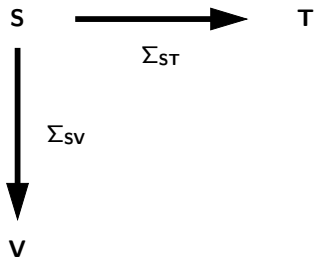
High-level algebraic operators, such as compose, are used to manipulate mappings.

- ▶ What other operators are needed?

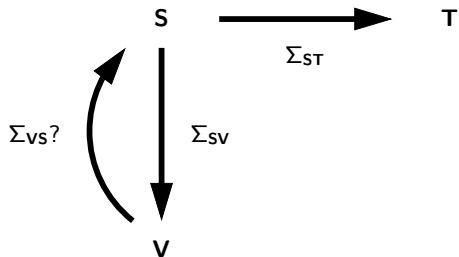
# An inverse operator is also needed



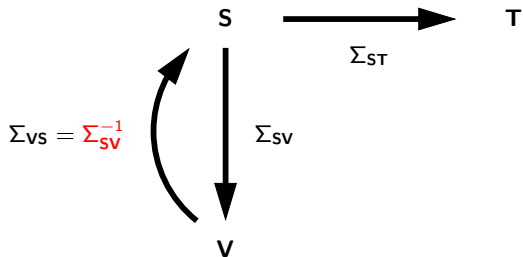
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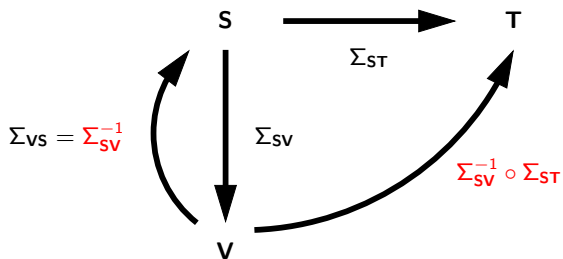
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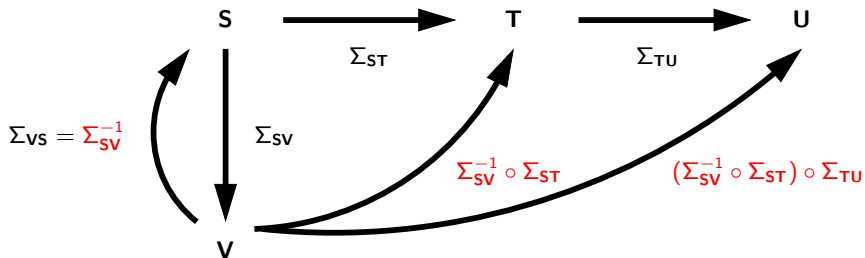


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# Metadata management: A more general data exchange framework is needed

Composition and inverse operators have been extensively studied in the relational world.

- ▶ Semantics, computation, ...

Combining these operators is an open issue.

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- ▶ Sources instances may contain null values

There is a need for a data exchange framework that can handle databases with **incomplete information**.

# But this is not the only reason . . .

Nowadays several applications use knowledge bases to represent data.

- ▶ A knowledge base has not only data but also **rules** that allows to infer new data
- ▶ In the Semantics Web: RDFS and OWL ontologies

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In a data exchange application over the Semantics Web:

The input is a mapping and a source specification including data and **rules**, and the output is a target specification also including data and **rules**

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The input is a mapping and a source specification including data and **rules**, and the output is a target specification also including data and **rules**

There is a need for a data exchange framework that can handle **knowledge bases**.

# Knowledge exchange: A more general data exchange framework is needed

## Example

Assume given the following source knowledge base:

Data:

Father		Mother	
Andy	Bob	Carrie	Bob
Bob	Danny		
Danny	Eddie		

Rules:

$$\begin{aligned} \text{Father}(x, y) &\rightarrow \text{Parent}(x, y) \\ \text{Mother}(x, y) &\rightarrow \text{Parent}(x, y) \\ \text{Parent}(x, y) \wedge \text{Parent}(y, z) &\rightarrow \text{Grandparent}(x, z) \end{aligned}$$



# Knowledge exchange: A more general data exchange framework is needed

## Example (cont'd)

Given a mapping:

$$\begin{aligned} \text{Father}(x, y) &\rightarrow \text{Padre}(x, y) \\ \text{Grandparent}(x, y) &\rightarrow \text{Abuelo}(x, y) \end{aligned}$$

What is a good translation of the initial knowledge base?

# Knowledge exchange: A more general data exchange framework is needed

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What is a good translation of the initial knowledge base?

Data:

Padre	
Andy	Bob
Bob	Danny
Danny	Eddie

Abuelo	
Andy	Danny
Carrie	Danny
Bob	Eddie

Rules:  $\emptyset$

# Knowledge exchange: A more general data exchange framework is needed

## Example (cont'd)

Our first alternative does not include any translation of the source rules:

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What data should we materialize?

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Abuelo	
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Is this a good translation? Why?

# One can exchange more than complete data

- ▶ In data exchange one starts with a database instance (with complete information).
- ▶ What if we have an initial object that has several interpretations?
  - ▶ A representation of a set of possible instances
- ▶ We propose a new general formalism to exchange representations of possible instances
  - ▶ We apply it to the problems of exchanging instances with incomplete information and exchanging knowledge bases

## Outline: Second part

- ▶ Formalism for exchanging representations systems
- ▶ Applications to incomplete instances
- ▶ Applications to knowledge bases
- ▶ Concluding remarks

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# Representation systems

A representation system  $\mathcal{R} = (\mathbf{W}, \text{rep})$  consists of:

- ▶ a set  $\mathbf{W}$  of *representatives*
- ▶ a function  $\text{rep}$  that assigns a set of instances to every element in  $\mathbf{W}$

$$\text{rep}(\mathcal{V}) = \{l_1, l_2, l_3, \dots\} \text{ for every } \mathcal{V} \in \mathbf{W}$$

Uniformity assumption: For every  $\mathcal{V} \in \mathbf{W}$ , there exists a relational schema  $\mathbf{S}$  (the type of  $\mathcal{V}$ ) such that  $\text{rep}(\mathcal{V}) \subseteq \text{Inst}(\mathbf{S})$

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Incomplete instances and knowledge bases are representation systems

# In classical data exchange we consider only *complete* data

Recall that given  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ ,  $I \in \text{Inst}(\mathbf{S})$  and  $J \in \text{Inst}(\mathbf{T})$ :  $J$  is a solution for  $I$  under  $\mathcal{M}$  if  $(I, J) \models \Sigma$

$$J \in \text{Sol}_{\mathcal{M}}(I)$$



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Recall that given  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ ,  $I \in \text{Inst}(\mathbf{S})$  and  $J \in \text{Inst}(\mathbf{T})$ :  $J$  is a solution for  $I$  under  $\mathcal{M}$  if  $(I, J) \models \Sigma$

$$J \in \text{Sol}_{\mathcal{M}}(I)$$

This can be extended to set of instances. Given  $\mathcal{X} \subseteq \text{Inst}(\mathbf{S})$ :

$$\text{Sol}_{\mathcal{M}}(\mathcal{X}) = \bigcup_{I \in \mathcal{X}} \text{Sol}_{\mathcal{M}}(I)$$

# Extending the definition to representation systems

Given:

- ▶ a mapping  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$
- ▶ a representation system  $\mathcal{R} = (\mathbf{W}, \text{rep})$
- ▶  $\mathcal{U}, \mathcal{V} \in \mathbf{W}$  of types  $\mathbf{S}$  and  $\mathbf{T}$ , respectively

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## Definition (APR11)

$\mathcal{V}$  is an  $\mathcal{R}$ -solution of  $\mathcal{U}$  under  $\mathcal{M}$  if

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Or equivalently:  $\mathcal{V}$  is an  $\mathcal{R}$ -solution of  $\mathcal{U}$  if for every  $J \in \text{rep}(\mathcal{V})$ , there exists  $I \in \text{rep}(\mathcal{U})$  such that  $J \in \text{Sol}_{\mathcal{M}}(I)$ .

What is a good solution in this framework?

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## Definition (APR11)

$\mathcal{V}$  is an *universal  $\mathcal{R}$ -solution* of  $\mathcal{U}$  under  $\mathcal{M}$  if

$$\text{rep}(\mathcal{V}) = \text{Sol}_{\mathcal{M}}(\text{rep}(\mathcal{U}))$$

# Strong representation systems

Let  $\mathcal{C}$  be a class of mappings.

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$\mathcal{R} = (\mathbf{W}, \text{rep})$  is a *strong representation system* for  $\mathcal{C}$  if for every  $\mathcal{M} \in \mathcal{C}$  and for every  $\mathcal{U} \in \mathbf{W}$ , there exists a  $\mathcal{V} \in \mathbf{W}$  :

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$$\text{rep}(\mathcal{V}) = \text{Sol}_{\mathcal{M}}(\text{rep}(\mathcal{U}))$$

If  $\mathcal{R} = (\mathbf{W}, \text{rep})$  is a strong representation system, then the universal solutions for the representatives in  $\mathbf{W}$  can be **represented** in the same system.

# Outline: Second part

- ▶ Formalism for exchanging representations systems
- ▶ Applications to incomplete instances
- ▶ Applications to knowledge bases
- ▶ Concluding remarks

# Motivating questions

What is a strong representation system for the class of mappings specified by st-tgds?

- ▶ Are instances including nulls enough?

Can the fundamental data exchange problems be solved in polynomial time in this setting?

- ▶ Computing (universal) solutions
- ▶ Computing certain answers

# Naive instances

We have already considered **naive instances**: Instances with null values

- ▶ Example: Canonical universal solution

A naive instance  $\mathcal{I}$  has labeled nulls:

$$R(1, n_1)$$

$$R(n_1, 2)$$

$$R(1, n_2)$$

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The interpretations of  $\mathcal{I}$  are constructed by replacing nulls by constants:

$$\text{rep}(\mathcal{I}) = \{K \mid \mu(\mathcal{I}) \subseteq K \text{ for some valuation } \mu\}$$



# Are naive instances expressive enough?

Naive instances have been extensively used in data exchange:

## Proposition (FKMP03)

*Let  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ , where  $\Sigma$  is a set of st-tgds. Then for every instance  $I$  of  $\mathbf{S}$ , there exists a naive instance  $\mathcal{J}$  of  $\mathbf{T}$  such that:*

$$\text{rep}(\mathcal{J}) = \text{Sol}_{\mathcal{M}}(I)$$

In fact, the canonical universal solution satisfies the property mentioned above.

# Are naive instances expressive enough?

But naive instances are not expressive enough to deal with incomplete information in the source instances:

## Proposition (APR11)

*Naive instances are not a strong representation system for the class of mappings specified by st-tgds*

# Are naive instances expressive enough?

## Example

Consider a mapping  $\mathcal{M}$  specified by:

$$\text{Manager}(x, y) \rightarrow \text{Reports}(x, y)$$
$$\text{Manager}(x, x) \rightarrow \text{SelfManager}(x)$$

The canonical universal solution for  $\mathcal{I} = \{\text{Manager}(n, \text{Peter})\}$  under  $\mathcal{M}$ :

$$\mathcal{J} = \{\text{Reports}(n, \text{Peter})\}$$

But  $\mathcal{J}$  is not a *good* solution for  $\mathcal{I}$ .

- ▶ It cannot represent the fact that if  $n$  is given value Peter, then  $\text{SelfManager}(\text{Peter})$  should hold in the target.

# Conditional instances

What should be added to naive instances to obtain a strong representation system?

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- ▶ Answer from database theory: Conditions on the nulls

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Conditional instances: Naive instances plus *tuple conditions*

A tuple condition is a positive Boolean combinations of:

- ▶ equalities and inequalities between nulls, and between nulls and constants

# Conditional instances

## Example

$$\begin{array}{l|l} R(1, n_1) & n_1 = n_2 \\ R(n_1, n_2) & n_1 \neq n_2 \vee n_2 = 2 \end{array}$$

# Conditional instances

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Semantics:



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$$\begin{array}{l|l} R(1, n_1) & n_1 = n_2 \\ R(n_1, n_2) & n_1 \neq n_2 \vee n_2 = 2 \end{array}$$

Semantics:

$$\underline{\mu(n_1) = \mu(n_2) = 2} \quad \underline{\mu(n_1) = \mu(n_2) = 3} \quad \underline{\mu(n_1) = 2, \mu(n_2) = 3}$$

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## Example

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Interpretations of a conditional instance  $\mathcal{I}$ :

$$\text{rep}(\mathcal{I}) = \{K \mid \mu(\mathcal{I}) \subseteq K \text{ for some valuation } \mu\}$$

# Positive conditional instances

Many problems are intractable over conditional instances.

- ▶ We also consider a restricted class of conditional instances

**Positive** conditional instances: Conditional instances without inequalities

# (Positive) conditional instances are enough

## Theorem (APR11)

*Both conditional instances and positive conditional instances are strong representation systems for the class of mappings specified by st-tgds.*

## Example

Consider again the mapping  $\mathcal{M}$  specified by:

$$\text{Manager}(x, y) \rightarrow \text{Reports}(x, y)$$
$$\text{Manager}(x, x) \rightarrow \text{SelfManager}(x)$$

The following is a universal solution for  $\mathcal{I} = \{\text{Manager}(n, \text{Peter})\}$

$\text{Reports}(n, \text{Peter})$	$\mid$	$true$
$\text{SelfManager}(\text{Peter})$	$\mid$	$n = \text{Peter}$

# Positive conditional instances are *exactly* the needed representation system

Positive conditional instances are *minimal*:

## Theorem (APR11)

*All the following are needed to obtain a strong representation system for the class of mappings specified by st-tgds:*

- ▶ *equalities between nulls*
- ▶ *equalities between constant and nulls*
- ▶ *conjunctions and disjunctions*

Conditional instances are enough but not minimal.



# Positive conditional instance can be used in practice!

Let  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ , where  $\Sigma$  is a set of st-tgds.

# Positive conditional instance can be used in practice!

Let  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ , where  $\Sigma$  is a set of st-tgds.

## Theorem (APR11)

*There exists a polynomial time algorithm that, given a positive conditional instance  $\mathcal{I}$  over  $\mathbf{S}$ , computes a positive conditional instance  $\mathcal{J}$  over  $\mathbf{T}$  that is a universal solution for  $\mathcal{I}$  under  $\mathcal{M}$ .*

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Let  $Q$  be a union of conjunctive queries over  $\mathbf{T}$ .

$$Q(\mathcal{J}) = \bigcap_{J \in \text{rep}(\mathcal{J})} Q(J)$$
$$\text{certain}_{\mathcal{M}}(Q, \mathcal{I}) = \bigcap_{\mathcal{J} \text{ is a solution for } \mathcal{I} \text{ under } \mathcal{M}} Q(\mathcal{J})$$

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The same result holds for the class of unions of conjunctive queries with at most one inequality per disjunct.

- ▶ The other important class of queries in the data exchange area for which certain answers can be computed in polynomial time

## Outline: Second part

- ▶ Formalism for exchanging representations systems
- ▶ Applications to incomplete instances
- ▶ Applications to knowledge bases
- ▶ Concluding remarks

# The semantics of *knowledge bases* is given by sets of instances

Knowledge base over  $\mathbf{S}$ :  $(I, \Gamma)$  such that

- ▶  $I \in \text{Inst}(\mathbf{S})$
- ▶  $\Gamma$  a set of rules over  $\mathbf{S}$

Semantics: finite models

$$\text{Mod}(I, \Gamma) = \{K \in \text{Inst}(\mathbf{S}) \mid I \subseteq K \text{ and } K \models \Gamma\}$$

## We can apply our formalism to knowledge bases

$(I_2, \Gamma_2)$  is a *KB-solution* for  $(I_1, \Gamma_1)$  under  $\mathcal{M}$  if:

$$\text{Mod}(I_2, \Gamma_2) \subseteq \text{Sol}_{\mathcal{M}}(\text{Mod}(I_1, \Gamma_1))$$

$(I_2, \Gamma_2)$  is a *universal KB-solution* for  $(I_1, \Gamma_1)$  under  $\mathcal{M}$  if:

$$\text{Mod}(I_2, \Gamma_2) = \text{Sol}_{\mathcal{M}}(\text{Mod}(I_1, \Gamma_1))$$



# Motivating questions

Same as for the case of instances with incomplete information.

- ▶ Constructing universal KB-solutions
- ▶ Answering target queries

New fundamental problem: Construct solutions including **as much implicit knowledge as possible**.

# What are good knowledge-base solutions?

First alternative: universal KB-solutions

But there exist some other KB-solutions desirable to materialize

- ▶ Minimality comes into play

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But there exist some other KB-solutions desirable to materialize

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Given sets  $\mathcal{X}$ ,  $\mathcal{Y}$  of instances:

- ▶  $\mathcal{X} \equiv_{\min} \mathcal{Y}$  if  $\mathcal{X}$  and  $\mathcal{Y}$  coincide in the minimal instances under  $\subseteq$

## Definition

$(I_2, \Gamma_2)$  is a *minimal KB-solution* of  $(I_1, \Gamma_1)$  under  $\mathcal{M}$  if:

$$\text{Mod}(I_2, \Gamma_2) \equiv_{\min} \text{Sol}_{\mathcal{M}}(\text{Mod}(I_1, \Gamma_1))$$

# Two requirements to construct minimal knowledge-base solutions

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## Definition

$\Gamma_2$  is safe for  $\Gamma_1$  and  $\mathcal{M}$ , if for every  $I_1$  there exists  $I_2$ :

$(I_2, \Gamma_2)$  is a minimal KB-solution of  $(I_1, \Gamma_1)$  under  $\mathcal{M}$

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## Definition

$\Gamma_2$  is *optimal-safe* if for every other safe set  $\Gamma'$ :

$$\Gamma_2 \models \Gamma'$$



# Computing minimal KB-solutions

To obtain algorithms for computing minimal KB-solutions, we need to specify the language used in knowledge bases.

- ▶ Full st-tgd:

$$\forall \bar{x} \forall \bar{y} (\varphi(\bar{x}, \bar{y}) \rightarrow \psi(\bar{x}))$$

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## Theorem (APR11)

*There exists a polynomial-time algorithm that, given  $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma)$ , where  $\Sigma$  is a set of full st-tgds, and given a set  $\Gamma_1$  of full tgds over  $\mathbf{S}$ , computes a set  $\Gamma_2$  of second-order logic sentences over  $\mathbf{T}$  that is optimal-safe for  $\Gamma_1$  and  $\mathcal{M}$ .*

# Computing minimal KB-solutions

Unfortunately, first-order logic is not expressive enough.

## Theorem (APR11)

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How can we deal with these problems in practice?

- ▶ We need to restrict the language used to specify knowledge bases: **Description logics!**

# Outline: Second part

- ▶ Formalism for exchanging representations systems
- ▶ Applications to incomplete instances
- ▶ Applications to knowledge bases
- ▶ Concluding remarks

# We can exchange more than complete data

We propose a general formalism to exchange *representation systems*

- ▶ Applications to incomplete instances
- ▶ Applications to knowledge bases

Next step: Apply our general setting to the Semantic Web

- ▶ Semantic Web data has *nulls* (blank nodes)
- ▶ Semantic Web specifications have rules (RDFS, OWL)

Lots of interesting problems to solve if knowledge bases are specified by means of description logics.

- ▶ Better results can be obtained

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# Thank you!

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